GCE214
Applied Mechanics-Statics

Lecture 06: 01/11/2017

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Class: Wednesday (3–5 pm)
Venue: LT1

Etiquettes and MOP

- Attendance is a requirement.
- There may be class assessments, during or after lecture.
- Computational software will be employed in solving problems
- Conceptual understanding will be tested
- Lively discussions are integral part of the lectures.

Lecture content

Simplifying Systems of Forces

- Reducing a system of forces to force-couple system
- Equivalent and equipollent systems of forces
- Further reduction of a system of forces

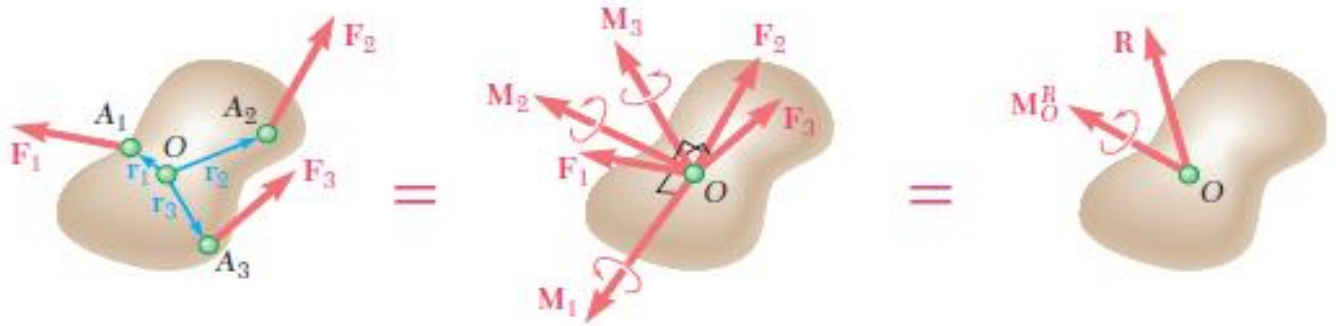
Recommended textbook

- Vector Mechanics for Engineers: Statics and Dynamics by Beer, Johnston, Mazurek, Cornwell. 10th Edition

SIMPLIFYING SYSTEMS OF FORCES

REDUCING A SYSTEM OF FORCES TO FORCE-COUPLE SYSTEM

- Considering a system of forces $\boldsymbol{F_1}, \boldsymbol{F_2}, \boldsymbol{F_3}\ldots$ acting on a rigid body at the points $A_1, A_2, A_3\ldots$, defined by position vectors $\boldsymbol{r_1}, \boldsymbol{r_2}, \boldsymbol{r_3}$ etc
- This system of forces can be reduced to an equivalent force-couple system acting at a given point *O*.



(*a*) (*b*) (*c*)

- The equivalent force-couple system is defined by

$$\mathbf{R} = \Sigma\mathbf{F} \qquad \mathbf{M}_O^R = \Sigma\mathbf{M}_O = \Sigma(\mathbf{r} \times \mathbf{F}) \qquad 1$$

- These equations state that we obtain force **R** by adding all of the forces of the system, whereas we obtain the moment of the resultant couple vector $\boldsymbol{M_O^R}$, called the **moment resultant** of the system, by adding the moments about *O* of all the forces of the system.

SIMPLIFYING SYSTEMS OF FORCES

REDUCING A SYSTEM OF FORCES TO FORCE-COUPLE SYSTEM

- In practice, the reduction of a given system of forces to a single force **R** at *O* and a couple vector $\boldsymbol{M}_O^R$ is carried out in terms of components (Eq 2).

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$
$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k} \qquad 2$$

- Substituting for **r** and **F** in Eq. (1) and factoring out the unit vectors **i, j,** and **k**, we obtain **R** and $\boldsymbol{M}_O^R$ in the form

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k} \qquad \mathbf{M}_O^R = M_x^R\mathbf{i} + M_y^R\mathbf{j} + M_z^R\mathbf{k} \qquad 3$$

- The components R_x, R_y, and R_z represent, respectively, the sums of the *x*, *y*, and *z* components of the given forces and measure ***the tendency of the system to impart to the rigid body a translation*** in the *x*, *y*, or *z* direction.
- Similarly, the components M_x^R, M_y^R, and M_Z^R represent, respectively, the sum of the moments of the given forces about the *x*, *y*, and *z* axes and measure ***the tendency of the system to impart to the rigid body a rotation*** about the *x*, *y*, or *z* axis.

SIMPLIFYING SYSTEMS OF FORCES

EQUIVALENT AND EQUIPOLLENT SYSTEMS OF FORCES

- Two systems of forces, F_1, F_2, F_3, . . . , and F_1', F_2', F_3' . . . , that act on the same rigid body are equivalent if, and only if, the sums of the forces and the sums of the moments about a given point *O* of the forces of the two systems are, respectively, equal.
- Mathematically, the necessary and sufficient conditions for the two systems of forces to be equivalent are

$$\Sigma \mathbf{F} = \Sigma \mathbf{F}' \quad \text{and} \quad \Sigma \mathbf{M}_O = \Sigma \mathbf{M}_O' \qquad 3$$

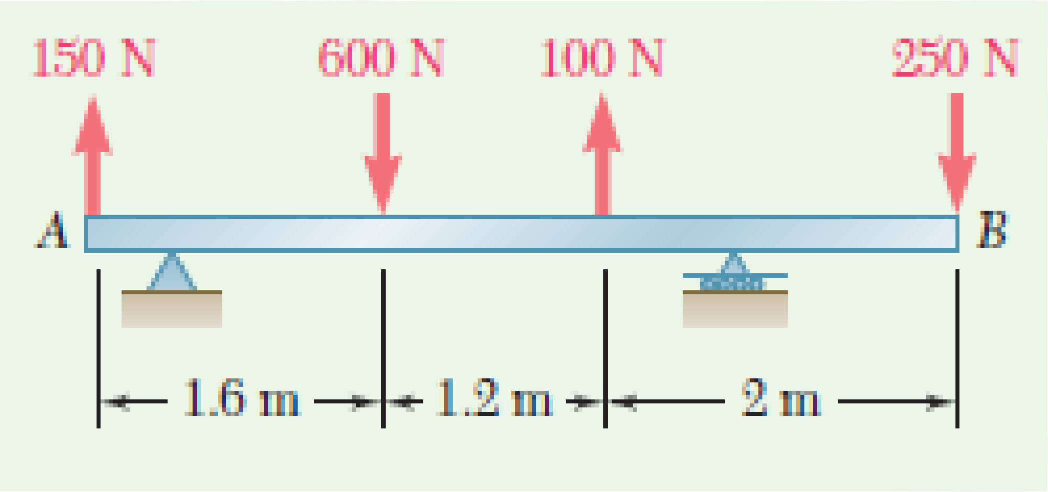
- In components form Eq. 3 yields

$$\begin{array}{lll} \Sigma F_x = \Sigma F_x' & \Sigma F_y = \Sigma F_y' & \Sigma F_z = \Sigma F_z' \\ \Sigma M_x = \Sigma M_x' & \Sigma M_y = \Sigma M_y' & \Sigma M_z = \Sigma M_z' \end{array} \qquad 4$$

- Two systems of forces are equivalent if they tend to impart to the rigid body (1) ***the same translation in the x, y, and z directions***, respectively, and (2) ***the same rotation about the x, y, and z axes***, respectively.
- In general, when two systems of vectors satisfy Eqs. (3) or (4), i.e., when their resultants and their moment resultants about an arbitrary point *O* are respectively equal, the two systems are said to be **equipollent.**

EXAMPLES

1. A 4.80-m-long beam is subjected to the forces shown. Reduce the given system of forces to (*a*) an equivalent force-couple system at *A*, *(b)* an equivalent force-couple system at *B*, *(c)* a single force or resultant. *Note:* Since the reactions at the supports are not included in the given system of forces, the given system will not maintain the beam in equilibrium.



SIMPLIFYING SYSTEMS OF FORCES

EXAMPLES

2. Four tugboats are bringing an ocean liner to its pier. Each tugboat exerts a force of 2270 N in the direction shown. Determine (*a*) the equivalent force-couple system at the foremast *O*, (*b*) the point on the hull where a single, more powerful tugboat should push to produce the same effect as the original four tugboats.